PROTOCOLLO
Proof Theory Track

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Main research topics

- Dynamics (of LL subsystems)
  - Correspondences between LL and λ-calculus (μ-calculus)
  - Shared-reductions and GOI
    theory and practice (parallel implementations)
- Sequent and term calculi for Intuitionistic and Classical Logic
  - Computational interpretation of Classical Logic
  - Polarized systems
  - Bi-intuitionistic logic
- Ludics

Main research topics (2)

- Proof-Nets
    - Correctness criteria
    - Complexity of correctness criteria
    - Interactive approach to correctness
    - Focalization and non-commutativity
- Non-commutative LL

Main research topics (3)

- λ-calculus
  - Reduction strategies (lazyness, cbv and cbn)
  - Intersection types
  - Separability
- Proof theory of modal logics
  - 2-sequents
  - Indexes

Workshop on Constructive CL

Held in Rome, April 2004
Guests:

- P.-L. Curien
- O. Laurent
- A. Saurin
- A. Guglielmi
- C. Faggian (involved in the new proposal)

PELCR

(Pedicini - Roma)

Parallel Environment for λ-calculus Optimal Reduction

- Based on a particular way to compute the GOI execution formula
- The key point is to make parallel execution effective
- It leads to several theoretical questions and open problems on how to further develop parallel implementations of optimal reduction

Bi-intuitionistic Logic

(Bellin, Fleury - Verona)

To reconsider Rauszer's bi-intuitionistic logic in the framework of the *logic of pragmatics*

Every formula is regarded as expressing an act of *assertion* or *conjecture*

- conjunction and implication are assertive
- subtraction and disjunction are conjectural

PBL

Polarized bi-intuitionistic logic (PBL) is the logic deriving from the previous approach.
It is formed of two fragments

- Positive intuitionistic logic (implication and conjunction)
- Dual intuitionistic logic (subtraction and disjunction)

Two negations ("certainly not" and "it is doubtful that") partially internalize the duality between the previous fragments.

Interpretations

Two modal interpretations and Kripke's semantics are considered, yielding two logics:

- one, still called PBL, interpreted on *bimodal preordered frames*
- the other, Intuitionistic Logic for Pragmatics of Assertions and Conjectures, interpreted over S4

Term assignment

It is a *calculus of continuations* exhibiting several features of calculi for concurrency, such as: broadcasting, remote binding and remote substitution.

The duality between the two fragments of PBL extends from formulas to proofs.

Every computation in the calculus of continuations is isomorphic to a computation in the simply typed λ-calculus.

Linear bi-intuitionistic logic

An analysis of bi-intuitionistic logic with the methods of Linear Logic.

- Syntax and semantics
- Tensor is monoidal and par is pre-monoidal
- The basis of a game semantics have been developed

Proofs-Tests as computations

(Guerrini - Roma; Masini - Verona)

Starting point: proof-test duality introduced by Girard in Meaning I.

a proof/test is a failure of a test/proof

- a complete deductive system for tests of intuitionistic logic
- tests correspond to continuations and proofs to programs
- a computation is the interaction between a continuation/test and a program/proof

CBV and CBN

The calculus has some nice properties:

- a redex is always at the root of the term
- reduction is a sort of interaction in the style of ludics or game semantics
- uniform treatment of CBV and CBN
- every term can be easily readback into a λ-term

Working in progress: extensions to classical logic (μ-calculus).

MALL Proof-Nets

(Maieli - Roma)

An interactive correctness criterion for unit-free multiplicative additive (MALL) proof-nets.

An extension of the interactive correctness criterion for MELL based on the interaction between proofs and para-proofs.

Exploiting the additive criterion for MALL given by Hughes and Van Glabeek.

Lazy Strong Normalization

(Pimentel, Ronchi, Roversi - Torino)

Logical characterization of lazy strongly β-normalizing terms using intersection types.

The characterization allows an interesting connection between cbv and cbn.

The class of lazy strongly β-normalizing terms coincides with that of cbv potentially valuable terms, that is a key notion for solvability in cbv.

PT of Intersection Types

(Paolini, Pimentel, Ronchi - Torino)

Derivations in Intersection Types (IT) form a strict subset of deductions in Intuitionistic Logic (LJ).

IT imposes a meta-theoretical restriction on proofs: conjunction can be introduced only between derivations that are synchronized with respect to the implication.

To present a proof-theoretical justification for IT.

ISL

To discuss the relationship between the intersection connective and the intuitionistic conjunction.

A new logical system called Intersection Synchronous Logic (ISL) is introduced.

ISL proves properties of sets of deductions of the implication-conjunction fragment of LJ.

Idea

The main idea behind ISL is the decomposition of the intuitionistic conjunction into two connectives, one with synchronous and the other with asynchronous behaviour.

ISL enjoys both

- strong normalization
- sub-formula properties

Future work

Proofs of ISL can be decorated with terms in a standard way, giving rise then to a calculus of processes, supporting both synchronous and asynchronous computations.

This decoration matches

- standard intersection type assignment system, when only the synchronous conjunction is taken into account
- the simple types assignment with pairs and projections, when the asynchronous conjunction is considered.

Working on an operational semantic for the calculus.

Separability

(Pagani - Roma)

Syntactic approach to the separability of (fragments) of LL.

Some negative results.

More in the presentation of “Semantics” track and in the talk.

Interaction Calculus

(Solitro - Verona)

A simple calculus (Interaction Calculus) for the representation of concurrent systems.

A system is a collection of expressions (processes) that share a working space; their computational behaviour is determined by the interaction of processes.

Computations are originated by a symmetric interaction between two expressions and carried out by substitutions

Functions can be nicely encoded.

Finitary proof theory of LTL

Reference:

A Proof-theoretic Investigation of a Logic of Positions

Annals of Pure and Applied Logic, vol. 123, 2003, pp. 135-162

Joint Work with S. Baratella, dip. Matematica, Trento.

Target:

Introduction of a normalizing extension of natural deduction that is suitable for Linear Temporal Logic (LTL) [modal operators plus induction].

The proposed calculus is strong normalizing.

the idea

Geometric extension of ordinary natural deduction by means of *position formulas* (A^s)

- A is a pure temporal formula
- $s = \langle n, S \rangle$ is the position of A (the place where A lives):
  - n is a natural number
  - S is a finite set of *tokens*. The use of tokens allows a precise definition of *modal interaction*.

examples of modal rules

$$\frac{\begin{matrix}\Gamma\\ \vdots \\ A^{\langle n,S\rangle}\end{matrix}}{\Box A^{\langle n,S\setminus\{x\}\rangle}}\ x\in S \text{ and } x\notin\Gamma$$

$$\frac{\begin{matrix}\vdots \\ \Box A^{\langle n,S\rangle}\end{matrix}}{A^{\langle n+m,S\cup T\rangle}}$$

$\langle m,T\rangle$ *is an arbitrary position.*

the induction rule

$$\dfrac{\begin{array}{cc} & [A^{\langle n, S\cup\{x\}\rangle}] \\ \vdots & \vdots \\ A^{\langle n,S\rangle} & A^{\langle n+1, S\cup\{x\}\rangle} \end{array}}{A^{\langle n+m, S\cup T\rangle}}$$

with the restriction that x *does not occur in S or in any of the assumptions on which the deduction of $A^{\langle n+1, S\cup\{x\}\rangle}$ depends, with the exception of the discharged assumptions $A^{\langle n, S\cup\{x\}\rangle}$*.

Infinitary proof theory of LTL

Reference:

An approach to infinitary temporal proof theory

Archive for Mathematical Logic (to appear)

Joint Work with S. Baratella, dip. Matematica, Trento

Target:

Investigation from a proof-theoretic viewpoint a propositional and a predicate sequent calculus with an ω–type schema of inference that naturally interpret the propositional and the predicate versions of **LTL** respectively.

Why an infinitary calculus ?

Finitary presentations of **LTL** suffer intrinsic limitations:

- lack of completeness in presence of an infinite set of extralogical axioms;
- lack of a recursive axiomatization for the first order predicate version.

A way to overcome those limitations is to allow infinitary rules

We propose to use indexed sequents:

$$A_1^{i_1}, \ldots, A_n^{i_n} \vdash B_1^{j_1}, \ldots, B_n^{j_n}$$

$$\frac{\Gamma, A^{i+1} \vdash \Delta}{\Gamma, \circ A^{i} \vdash \Delta} \quad \circ\vdash \qquad\qquad \frac{\Gamma \vdash A^{i+1}, \Delta}{\Gamma \vdash \circ A^{i}, \Delta} \quad \vdash\circ$$

$$\frac{\Gamma, A^{i+k} \vdash \Delta}{\Gamma, \Box A^{i} \vdash \Delta} \quad \Box\vdash \qquad\qquad \frac{\{\Gamma \vdash A^{i+j}, \Delta\}_{j \in \omega}}{\Gamma \vdash \Box A^{i}, \Delta} \quad \vdash\Box$$

Theorem

The $\omega-$ sequent calculus enjoys *full* cut elimination

Infinitary proof theory for MTL

Reference:

An infinitary variant of Metric Temporal Logic over dense time domains

Mathematical Logic Quarterly, vol. 50, 2004, pp. 249-257

Joint Work with S. Baratella, dip. Matematica, Trento

Target:

Introduction of a complete and cut-free proof system for a sufficiently expressive fragment of Metric Temporal Logic over dense time domains in which a schema of induction is provable.

MTL

- **MTL** = **M**etric **T**emporal **L**ogic:
  a "quantitative" modal logic suitable to formalize and to prove properties of time critical systems
- a proof-theoretic investigation of **MTL** has been only partly attempted so far
- **MTL** is mainly used by means of model and automata theoretic methods

dense domain

The case of *dense time* is difficult!

The basic operator:
$\Box_{[m,n]}A$: *the formula A is true in the interval* $[f(m), f(n)]$
(f is a map from natural numbers to values of a dense domain)

The main problem is in "completeness".

an infinitary solution

A dose of infinity helps!

The recipe:

1. Extend sequents by means of labelled formulas $(t : A)$
2. use both a relational and a temporal language;
3. use $L_{\omega_1\omega}$ (only for the relational part).

$$\frac{\{\Gamma, t < f^n(0) \vdash \Delta\}_{n\in\omega}}{\Gamma, \bigvee_{n\in\omega} (t < f^n(0)) \vdash \Delta}$$

$$\frac{\Gamma \vdash t < f^m(0), \Delta}{\Gamma \vdash \bigvee_{n\in\omega} (t < f^n(0)), \Delta} \quad \text{for all } m \in \omega.$$

The rules for temporal operator are “strongly inspired” by rules of bounded arithmetic:

$$\frac{\Gamma, f^m(t) \leq x, x \leq f^n(t) \vdash x : \alpha, \Delta}{\Gamma \vdash t : \Box_{[m,n]}\alpha, \Delta}$$

where x does not occur free in $\Gamma \cup \Delta \cup \{t\}$